Section A Part 1 (No Calculator)

1 Consider the quadratic function $y = -2(x+2)(x-1)$.

a State the x-intercepts.
b State the equation of the axis of symmetry.
c Find the y-intercept.
d Find the coordinates of the vertex.
e Sketch the function.

2 Solve the following equations, giving exact answers:

a $3x^2 - 12x = 0$ **b** $3x^2 - x - 10 = 0$ **c** $x^2 - 11x = 60$

3 Solve using the quadratic formula:

a $x^2 + 5x + 3 = 0$ **b** $3x^2 + 11x - 2 = 0$

4 Solve by 'completing the square': $x^2 + 7x - 4 = 0$

5 Use the vertex, axis of symmetry, and y-intercept to graph:

a $y = (x-2)^2 - 4$ **b** $y = -\frac{1}{2}(x+4)^2 + 6$

6 Find, in the form $y = ax^2 + bx + c$, the equation of the quadratic whose graph:

a touches the x-axis at 4 and passes through $(2, 12)$
b has vertex $(-4, 1)$ and passes through $(1, 11)$.

7 Find the maximum or minimum value of the relation $y = -2x^2 + 4x + 3$ and the value of x at which this occurs.

8 The roots of $2x^2 - 3x = 4$ are α and β. Find the simplest quadratic equation which has roots $\frac{1}{\alpha}$ and $\frac{1}{\beta}$.

9 Solve the following equations:

a $x^2 + 10 = 7x$ **b** $x + \frac{12}{x} = 7$ **c** $2x^2 - 7x + 3 = 0$

10 Find the points of intersection of $y = x^2 - 3x$ and $y = 3x^2 - 5x - 24$.

11 For what values of k does the graph of $y = -2x^2 + 5x + k$ not cut the x-axis?

12 Find the values of m for which $2x^2 - 3x + m = 0$ has:

a a repeated root **b** two distinct real roots **c** no real roots.

13 The sum of a number and its reciprocal is $2\frac{1}{30}$. Find the number.

14 Show that no line with a y-intercept of $(0, 10)$ will ever be tangential to the curve with equation $y = 3x^2 + 7x - 2$.

15 One of the roots of $kx^2 + (1-3k)x + (k-6) = 0$ is the negative reciprocal of the other root. Find k and the two roots.

Section A Part 2 (Calculator)

1 Consider the quadratic function $y = 2x^2 + 6x - 3$.

a Convert it to the form $y = a(x-h)^2 + k$.
b State the coordinates of the vertex.
c Find the y-intercept.
d Sketch the graph of the function.

2 Solve:

a $(x-2)(x+1)=3x-4$

b $2x-\frac{1}{x}=5$

3 Draw the graph of $y=-x^2+2x$.

4 Consider the quadratic function $y=-3x^2+8x+7$. Find the equation of the axis of symmetry, and the coordinates of the vertex.

5 Using the discriminant only, determine the nature of the solutions of:

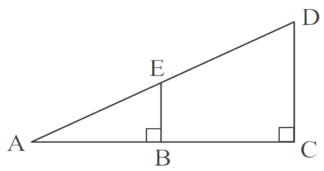
a $2x^2-5x-7=0$

b $3x^2-24x+48=0$

6 **a** For what values of c do the lines with equations $y=3x+c$ intersect the parabola $y=x^2+x-5$ in two distinct points?

b Choose one such value of c from part **a** and find the points of intersection in this case.

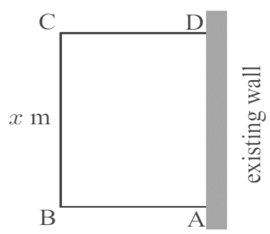
7 Suppose [AB] has the same length as [CD], [BC] is 2 cm shorter than [AB], and [BE] is 7 cm in length. Find the length of [AB].



8 60 m of chicken wire is available to construct a rectangular chicken enclosure against an existing wall.

a If $BC = x$ m, show that the area of rectangle ABCD is given by $A=(30x-\frac{1}{2}x^2)$ m^2.

b Find the dimensions of the enclosure which will maximise the area enclosed.



9 Consider the quadratic function $y=2x^2+4x-1$.

a State the axis of symmetry.

b Find the coordinates of the vertex.

c Find the axes intercepts.

d Hence sketch the function.

10 An open square-based container is made by cutting 4 cm square pieces out of a piece of tinplate. If the volume of the container is 120 cm^3, find the size of the original piece of tinplate.

11 Consider $y=-x^2-5x+3$ and $y=x^2+3x+11$.

a Solve for x: $-x^2-5x+3=x^2+3x+11$.

b Hence, or otherwise, determine the values of x for which $x^2+3x+11>-x^2-5x+3$.

12 Find the maximum or minimum value of the following quadratics, and the corresponding value of x:

a $y=3x^2+4x+7$

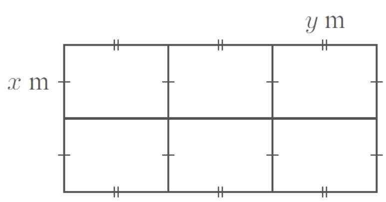
b $y=-2x^2-5x+2$

13 600 m of fencing is used to construct 6 rectangular animal pens as shown.

a Show that the area A of each pen is $A=x\left(\frac{600-8x}{9}\right)$ m^2.

b Find the dimensions of each pen so that it has the maximum possible area.

c What is the area of each pen in this case?



14 Two different quadratic functions of the form $y=9x^2-kx+4$ each touch the x-axis.

a Find the two values of k.

b Find the point of intersection of the two quadratic functions.

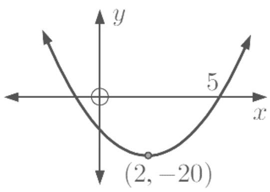
Section A Part 3 (Calculator)

3 Solve the following using the quadratic formula:

a $x^2 - 7x + 3 = 0$ **b** $2x^2 - 5x + 4 = 0$

4 Find the equation of the quadratic function with graph:

a



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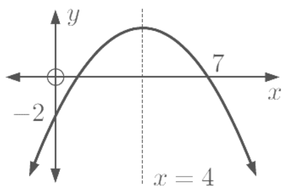

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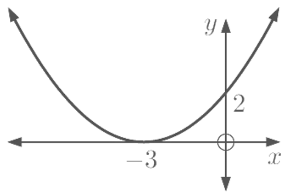


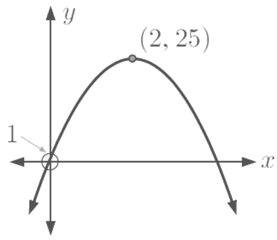
5 Use the discriminant only to find the relationship between the graph and the x-axis for:

a $y = 2x^2 + 3x - 7$ **b** $y = -3x^2 - 7x + 4$

6 Determine whether the following quadratic functions are positive definite, negative definite, or neither:

a $y = -2x^2 + 3x + 2$ **b** $y = 3x^2 + x + 11$

7 Find the equation of the quadratic function shown:



8 Find the y-intercept of the line with gradient -3 that is tangential to the parabola $y = 2x^2 - 5x + 1$.

9 For what values of k would the graph of $y = x^2 - 2x + k$ cut the x-axis twice?

10 Find the quadratic function which cuts the x-axis at 3 and -2 and which has y-intercept 24. Give your answer in the form $y = ax^2 + bx + c$.

11 For what values of m are the lines $y = mx - 10$ tangents to the parabola $y = 3x^2 + 7x + 2$?

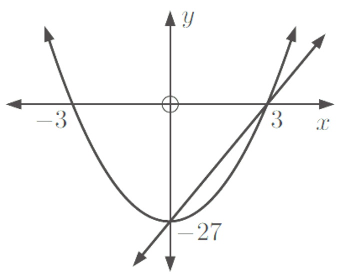
12 $ax^2 + (3 - a)x - 4 = 0$ has roots which are real and positive. What values can a have?

13 **a** Determine the equation of:

i the quadratic function

ii the straight line.

b For what values of x is the straight line above the curve?



14 Show that the lines with equations $y = -5x + k$ are tangents to the parabola $y = x^2 - 3x + c$ if and only if $c - k = 1$.

15 $4x^2 - 3x - 3 = 0$ has roots p, q. Find *all* quadratic equations with roots p^3 and q^3.

Section B , Part 1 (No Calculator)

1 If $f(x) = x^2 - 2x$, find in simplest form:

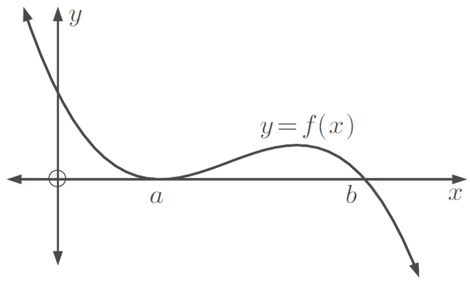
a $f(3)$ **b** $f(2x)$ **c** $f(-x)$ **d** $3f(x) - 2$

2 If $f(x) = 5 - x - x^2$, find in simplest form:

a $f(-1)$ **b** $f(x-1)$ **c** $f\left(\frac{x}{2}\right)$ **d** $2f(x) - f(-x)$

3 The graph of $f(x) = 3x^3 - 2x^2 + x + 2$ is translated to its image $g(x)$ by the vector $\binom{1}{-2}$. Write the equation of $g(x)$ in the form $g(x) = ax^3 + bx^2 + cx + d$.

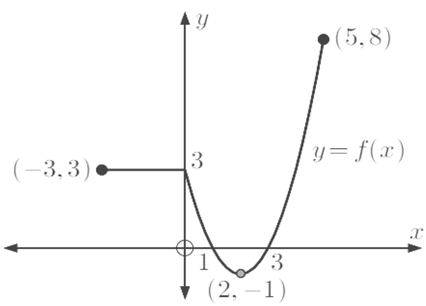
4 The graph of $y = f(x)$ is shown alongside. The x-axis is a tangent to $f(x)$ at $x = a$ and $f(x)$ cuts the x-axis at $x = b$. On the same diagram, sketch the graph of $y = f(x - c)$ where $0 < c < b - a$. Indicate the x-intercepts of $y = f(x - c)$.



5 For the graph of $y = f(x)$ given, sketch graphs of:

a $y = f(-x)$ **b** $y = -f(x)$

c $y = f(x+2)$ **d** $y = f(x) + 2$



6 Consider the function $f : x \mapsto x^2$. On the same set of axes graph:

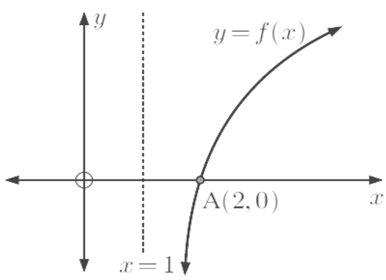
a $y = f(x)$ **b** $y = f(x-1)$ **c** $y = 3f(x-1)$ **d** $y = 3f(x-1) + 2$

7 The graph of $y = f(x)$ is shown alongside.

a Sketch the graph of $y = g(x)$ where $g(x) = f(x+3) - 1$.

b State the equation of the vertical asymptote of $y = g(x)$.

c Identify the point A$'$ on the graph of $y = g(x)$ which corresponds to point A.



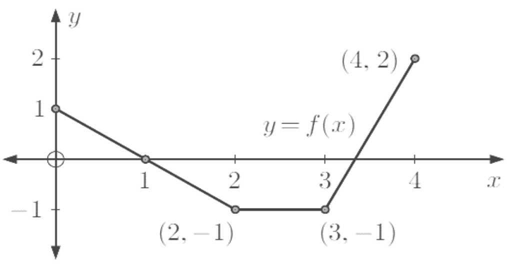
8 The graph of $y = f(x)$ is drawn alongside.

a Draw the graphs of $y = f(x)$ and $y = |f(x)|$ on the same set of axes.

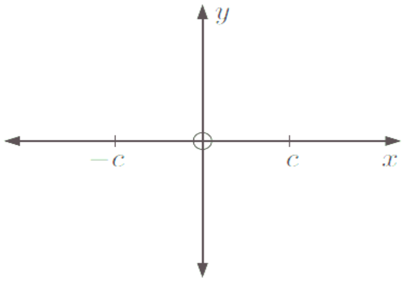
b Find the y-intercept of $\frac{1}{f(x)}$.

c Show on the diagram the points that are invariant for the function $\frac{1}{f(x)}$.

d Draw the graphs of $y = f(x)$ and $y = \frac{1}{f(x)}$ on the same set of axes.



9 Let $f(x) = \frac{c}{x+c}$, $x \neq -c$, $c > 0$.



a On a set of axes like those shown, sketch the graph of $y = f(x)$. Label clearly any points of intersection with the axes and any asymptotes.

b On the same set of axes, sketch the graph of $y = \frac{1}{f(x)}$. Label clearly any points of intersection with the axes.

10 Consider $f(x) = x - a$ where a is a positive real number.

a Find expressions for $|f(x)|$ and $f(|x|)$.

b Sketch $y = |f(x)|$ and $y = f(|x|)$ on the same set of axes.

c Solve for x given a is a positive real number: $|x - a| = |x| - a$.

Section B, Part 2 (Calculator)

1 Use your calculator to help graph $f(x) = (x+1)^2 - 4$. Include all axes intercepts, and the coordinates of the turning point of the function.

2 Consider the function $f : x \mapsto x^2$. On the same set of axes graph:

a $y = f(x)$ **b** $y = f(x+2)$ **c** $y = 2f(x+2)$ **d** $y = 2f(x+2) - 3$

3 Consider $f : x \mapsto \frac{2^x}{x}$.

a Does the function have any axes intercepts?

b Find the equations of the asymptotes of the function.

c Find any turning points of the function.

d Sketch the function for $-4 \leqslant x \leqslant 4$.

4 Consider $f(x) = 2^{-x}$.

a Use your calculator to help determine whether the following are true or false:

i As $x \to \infty$, $2^{-x} \to 0$. **ii** As $x \to -\infty$, $2^{-x} \to 0$.

iii The y-intercept is $\frac{1}{2}$. **iv** $2^{-x} > 0$ for all x.

b On the same set of axes, graph $y = f(x)$ and $y = |f(x)|$.

c Write down the equation of any asympotes of $y = |f(x)|$.

5 The graph of the function $f(x) = (x+1)^2 + 4$ is translated 2 units to the right and 4 units up.

a Find the function $g(x)$ corresponding to the translated graph.

b State the range of $f(x)$.

c State the range of $g(x)$.

6 For each of the following functions:

i Find $y = f(x)$, the result when the function is translated by $\binom{1}{-2}$.

ii Sketch the original function and its translated function on the same set of axes. Clearly state any asymptotes of each function.

iii State the domain and range of each function.

a $y = \frac{1}{x}$ **b** $y = 2^x$

7 Sketch the graph of $f(x) = x^2 + 1$, and on the same set of axes sketch the graphs of:

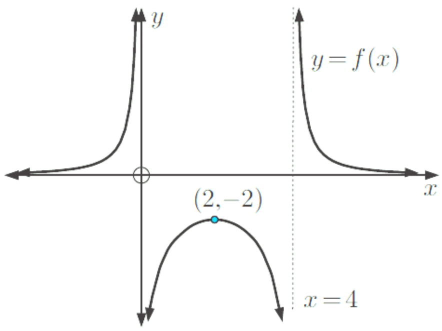
a $-f(x)$ **b** $f(2x)$ **c** $f(x) + 3$

8 Suppose $f(x) = x + 2$. The function F is obtained by stretching the function f vertically with scale factor 2, then stretching it horizontally with scale factor $\frac{1}{2}$, then translating it $\frac{1}{2}$ horizontally and -3 vertically.

- **a** Find the function $F(x)$.
- **b** What can be said about the point $(1, 3)$ under this transformation?
- **c** What happens to the points $(0, 2)$ and $(-1, 1)$ under this transformation?
- **d** Show that the points in **c** also lie on the graph of $y = F(x)$.

9 The graph of $y = f(x)$ is given.
On the same set of axes graph each pair of functions:

- **a** $y = f(x)$ and $y = f(x-2) + 1$
- **b** $y = f(x)$ and $y = \dfrac{1}{f(x)}$
- **c** $y = f(x)$ and $y = |f(x)|$



10 Consider the function $f(x) = \dfrac{2x-3}{3x+5}$.

- **a** Find the asymptotes of $y = f(x)$.
- **b** Discuss the behaviour of the graph near these asymptotes.
- **c** Find the axes intercepts of $y = f(x)$.
- **d** Sketch the graph of $y = f(x)$.
- **e** Describe the transformations which transform $y = \dfrac{1}{x}$ into $y = f(x)$.
- **f** Describe the transformations which transform $y = f(x)$ into $y = \dfrac{1}{x}$.

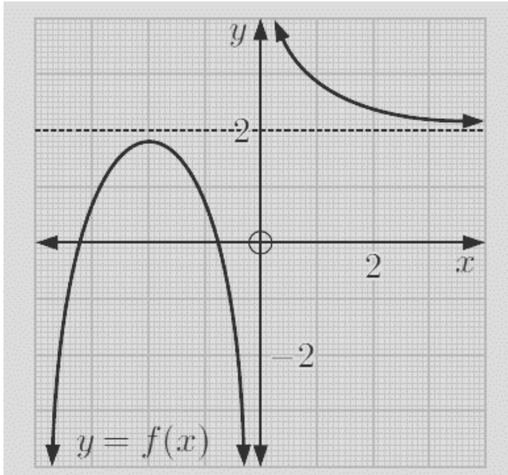
11
- **a** Sketch the graph of $f(x) = -2x + 3$, clearly showing the axes intercepts.
- **b** Find the invariant points for the graph of $y = \dfrac{1}{f(x)}$.
- **c** State the equation of the vertical asymptote of $y = \dfrac{1}{f(x)}$ and find its y-intercept.
- **d** Sketch the graph of $y = \dfrac{1}{f(x)}$ on the same axes as in part **a**, showing clearly the information you have found.
- **e** On a new pair of axes, sketch the graphs of $y = |f(x)|$ and $y = f(|x|)$ showing clearly all important features.

Section B, Part 3 (Calculator)

1 If $f(x) = \dfrac{4}{x}$, find in simplest form:

a $f(-4)$ **b** $f(2x)$ **c** $f\left(\dfrac{x}{2}\right)$ **d** $4f(x+2) - 3$

2 Consider the graph of $y = f(x)$ shown.



a Use the graph to determine:

i the coordinates of the turning point

ii the equation of the vertical asymptote

iii the equation of the horizontal asymptote

iv the x-intercepts.

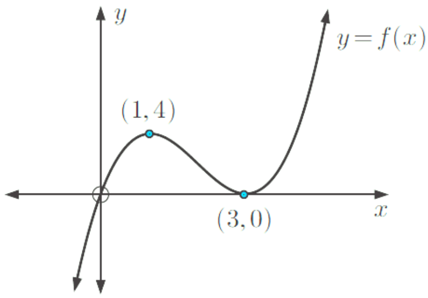
b Graph the function $g : x \mapsto x+1$ on the same set of axes.

c Hence estimate the coordinates of the points of intersection of $y = f(x)$ and $y = g(x)$.

3 Sketch the graph of $f(x) = -x^2$, and on the same set of axes sketch the graph of:

a $y = f(-x)$ **b** $y = -f(x)$ **c** $y = f(2x)$ **d** $y = f(x-2)$

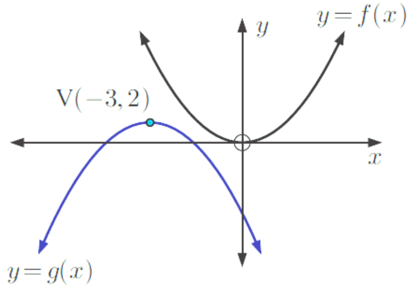
4 The graph of a cubic function $y = f(x)$ is shown alongside.



a Sketch the graph of $g(x) = -f(x-1)$.

b State the coordinates of the turning points of $y = g(x)$.

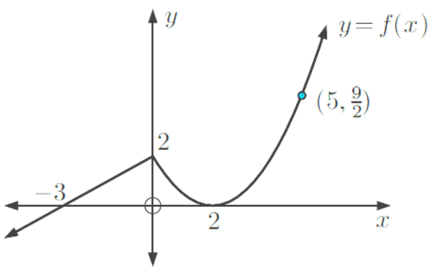
5 The graph of $f(x) = x^2$ is transformed to the graph of $g(x)$ by a reflection and a translation as illustrated. Find the formula for $g(x)$ in the form $g(x) = ax^2 + bx + c$.



6 Given the graph of $y = f(x)$, sketch graphs of:

a $f(-x)$ **b** $f(x+1)$

c $f(x) - 3$



7 The graph of $f(x) = x^3 + 3x^2 - x + 4$ is translated to its image $y = g(x)$ by the vector $\binom{-1}{3}$. Write the equation of $g(x)$ in the form $g(x) = ax^3 + bx^2 + cx + d$.

8 **a** Find the equation of the line that results when the line $f(x) = 3x + 2$ is translated:

i 2 units to the left **ii** 6 units upwards.

b Show that when the linear function $f(x) = ax + b$, $a > 0$ is translated k units to the left, the resulting line is the same as when $f(x)$ is translated ka units upwards.

9 The function $f(x)$ results from transforming the function $y = \frac{1}{x}$ by a reflection in the y-axis, then a vertical stretch with scale factor 3, then a translation of $\binom{1}{1}$.

a Find an expression for $f(x)$.

b Sketch $y = f(x)$ and state its domain and range.

c Does $y = f(x)$ have an inverse function? Explain your answer.

d Is the function f a self-inverse function? Give graphical and algebraic evidence to support your answer.

10 Consider $y = \log_4 x$.

a Find the function which results from a translation of $\binom{1}{-2}$.

b Sketch the original function and the translated function on the same set of axes.

c State the asymptotes of each function.

d State the domain and range of each function.

11 The function $g(x)$ results when $y = \frac{1}{x}$ is transformed by a vertical stretch with scale factor $\frac{1}{3}$, followed by a reflection in the y-axis, followed by a translation of 2 units to the right.

a Write an expression for $g(x)$ in the form $g(x) = \frac{ax + b}{cx + d}$.

b Find the asymptotes of $y = g(x)$.

c State the domain and range of $g(x)$.

d Sketch $y = g(x)$.

Section C , Part 1 (No Calculator)

1 Simplify:

a $-(-1)^{10}$ **b** $-(-3)^3$ **c** $3^0 - 3^{-1}$

2 Simplify using the laws of exponents:

a $a^4b^5 \times a^2b^2$ **b** $6xy^5 \div 9x^2y^5$ **c** $\frac{5(x^2y)^2}{(5x^2)^2}$

3 Let $f(x) = 3^x$.

a Write down the value of: **i** $f(4)$ **ii** $f(-1)$

b Find the value of k such that $f(x+2) = k\,f(x)$, $k \in \mathbb{Z}$.

4 Write without brackets or negative exponents:

a $x^{-2} \times x^{-3}$ **b** $2(ab)^{-2}$ **c** $2ab^{-2}$

5 Write as a single power of 3:

a $\frac{27}{9^a}$ **b** $(\sqrt{3})^{1-x} \times 9^{1-2x}$

6 Evaluate:

a $8^{\frac{2}{3}}$ **b** $27^{-\frac{2}{3}}$

7 Write without negative exponents:

a mn^{-2} **b** $(mn)^{-3}$ **c** $\frac{m^2n^{-1}}{p^{-2}}$ **d** $(4m^{-1}n)^2$

8 Expand and simplify:

a $(3 - e^x)^2$ **b** $(\sqrt{x}+2)(\sqrt{x}-2)$ **c** $2^{-x}(2^{2x} + 2^x)$

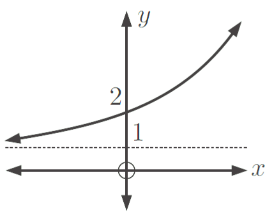
9 Find the value of x:

a $2^{x-3} = \frac{1}{32}$ **b** $9^x = 27^{2-2x}$ **c** $e^{2x} = \frac{1}{\sqrt{e}}$

10 Match each equation to its corresponding graph:

a $y = -e^x$ **b** $y = 3 \times 2^x$ **c** $y = e^x + 1$ **d** $y = 3^{-x}$ **e** $y = -e^{-x}$

A



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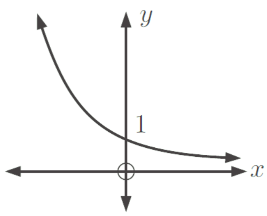

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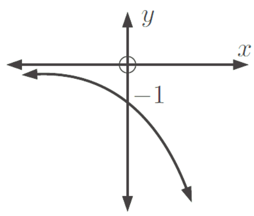

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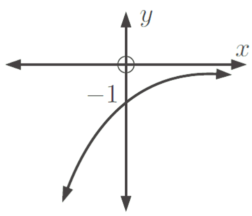

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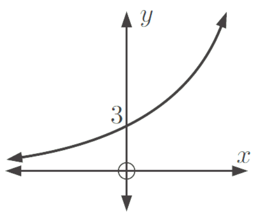


11 Suppose $y = a^x$. Express in terms of y:

a a^{2x} **b** a^{-x} **c** $\frac{1}{\sqrt{a^x}}$

Section C, Part 2 (No Calculator)

1 **a** Write 4×2^n as a power of 2.
b Evaluate $7^{-1} - 7^0$.
c Write $(\frac{2}{3})^{-3}$ in simplest fractional form.
d Write $\left(\frac{2a^{-1}}{b^2}\right)^2$ without negative exponents or brackets.

2 Evaluate, correct to 3 significant figures:
a $3^{\frac{3}{4}}$ **b** $27^{-\frac{1}{5}}$ **c** $\sqrt[4]{100}$

3 If $f(x) = 3 \times 2^x$, find the value of:
a $f(0)$ **b** $f(3)$ **c** $f(-2)$

4 Suppose $f(x) = 2^{-x} + 1$.
a Find $f(\frac{1}{2})$. **b** Find a such that $f(a) = 3$.

5 On the same set of axes draw the graphs of $y = 2^x$ and $y = 2^x - 4$. Include on your graph the y-intercept and the equation of the horizontal asymptote of each function.

6 The temperature of a dish t minutes after it is removed from the microwave, is given by $T = 80 \times (0.913)^t$ °C.
a Find the initial temperature of the dish.
b Find the temperature after:
i $t = 12$ **ii** $t = 24$ **iii** $t = 36$ minutes.
c Draw the graph of T against t for $t \geqslant 0$, using the above or technology.
d Hence, find the time taken for the temperature of the dish to fall to 25°C.

7 Consider $y = 3^x - 5$.
a Find y when $x = 0, \pm 1, \pm 2$. **b** Discuss y as $x \to \pm\infty$.
c Sketch the graph of $y = 3^x - 5$. **d** State the equation of any asymptote.

8 **a** On the same set of axes, sketch and clearly label the graphs of:
$f : x \mapsto e^x$, $g : x \mapsto e^{x-1}$, $h : x \mapsto 3 - e^x$
b State the domain and range of each function in **a**.

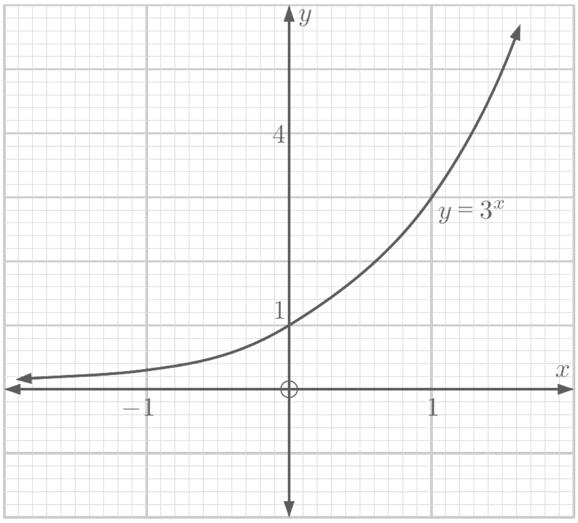
9 Consider $y = 3 - 2^{-x}$.
a Find y when $x = 0, \pm 1, \pm 2$. **b** Discuss y as $x \to \pm\infty$.
c Sketch the graph of $y = 3 - 2^{-x}$. **d** State the equation of any asymptote.

10 The weight of a radioactive substance after t years is given by $W = 1500 \times (0.993)^t$ grams.
a Find the original amount of radioactive material.
b Find the amount of radioactive material remaining after:
i 400 years **ii** 800 years.
c Sketch the graph of W against t, $t \geqslant 0$, using the above or technology.
d Hence, find the time taken for the weight to reduce to 100 grams.

Section C, Part 3 (No Calculator)

1 Given the graph of $y = 3^x$ shown, estimate solutions to the exponential equations:

a $3^x = 5$
b $3^x = \frac{1}{2}$
c $6 \times 3^x = 20$



2 Simplify using the laws of exponents:

a $(a^7)^3$ **b** $pq^2 \times p^3q^4$ **c** $\frac{8ab^5}{2a^4b^4}$

3 Write the following as a power of 2:

a 2×2^{-4} **b** $16 \div 2^{-3}$ **c** 8^4

4 Write without brackets or negative exponents:

a b^{-3} **b** $(ab)^{-1}$ **c** ab^{-1}

5 Simplify $\frac{2^{x+1}}{2^{1-x}}$.

6 Write as powers of 5 in simplest form:

a 1 **b** $5\sqrt{5}$ **c** $\frac{1}{\sqrt[4]{5}}$ **d** 25^{a+3}

7 Expand and simplify:

a $e^x(e^{-x} + e^x)$ **b** $(2^x + 5)^2$ **c** $(x^{\frac{1}{2}} - 7)(x^{\frac{1}{2}} + 7)$

8 Solve for x:

a $6 \times 2^x = 192$ **b** $4 \times (\frac{1}{3})^x = 324$

9 The point $(1, \sqrt{8})$ lies on the graph of $y = 2^{kx}$. Find the value of k.

10 Solve for x without using a calculator:

a $2^{x+1} = 32$ **b** $4^{x+1} = \left(\frac{1}{8}\right)^x$

11 Consider $y = 2e^{-x} + 1$.

a Find y when $x = 0, \pm 1, \pm 2$.
b Discuss y as $x \to \pm\infty$.
c Sketch the graph of $y = 2e^{-x} + 1$.
d State the equation of any asymptote.

Section D , Part 1 (No Calculator)

1 Find the following, showing all working.

a $\log_4 64$ **b** $\log_2 256$ **c** $\log_2(0.25)$ **d** $\log_{25} 5$

e $\log_8 1$ **f** $\log_{81} 3$ **g** $\log_9(0.\overline{1})$ **h** $\log_k \sqrt{k}$

2 Find:

a $\log \sqrt{10}$ **b** $\log \frac{1}{\sqrt[3]{10}}$ **c** $\log(10^a \times 10^{b+1})$

3 Simplify:

a $4\ln 2 + 2\ln 3$ **b** $\frac{1}{2}\ln 9 - \ln 2$ **c** $2\ln 5 - 1$ **d** $\frac{1}{4}\ln 81$

4 Find:

a $\ln(e\sqrt{e})$ **b** $\ln\left(\frac{1}{e^3}\right)$ **c** $\ln(e^{2x})$ **d** $\ln\left(\frac{e}{e^x}\right)$

5 Write as a single logarithm:

a $\log 16 + 2\log 3$ **b** $\log_2 16 - 2\log_2 3$ **c** $2 + \log_4 5$

6 Write as logarithmic equations:

a $P = 3 \times b^x$ **b** $m = \frac{n^3}{p^2}$

7 Show that $\log_3 7 \times 2\log_7 x = 2\log_3 x$.

8 Write the following equations without logarithms:

a $\log T = 2\log x - \log y$ **b** $\log_2 K = \log_2 n + \frac{1}{2}\log_2 t$

9 Write in the form $a \ln k$ where a and k are positive whole numbers and k is prime:

a $\ln 32$ **b** $\ln 125$ **c** $\ln 729$

10 Copy and complete:

Function	$y = \log_2 x$	$y = \ln(x+5)$
Domain		
Range		

11 If $A = \log_5 2$ and $B = \log_5 3$, write in terms of A and B:

a $\log_5 36$ **b** $\log_5 54$ **c** $\log_5(8\sqrt{3})$ **d** $\log_5(20.25)$ **e** $\log_5(0.\overline{8})$

12 Solve for x:

a $3e^x - 5 = -2e^{-x}$ **b** $2\ln x - 3\ln\left(\frac{1}{x}\right) = 10$

Section D , Part 2 (Calculator)

1 Write in the form 10^x giving x correct to 4 decimal places:

a 32 **b** 0.0013 **c** 8.963×10^{-5}

2 Find x if:

a $\log_2 x = -3$ **b** $\log_5 x \approx 2.743$ **c** $\log_3 x \approx -3.145$

3 Write the following equations without logarithms:

a $\log_2 k \approx 1.699 + x$ **b** $\log_a Q = 3\log_a P + \log_a R$

c $\log A \approx 5\log B - 2.602$

4 Solve for x, giving exact answers:

a $5^x = 7$ **b** $20 \times 2^{2x+1} = 640$

5 The weight of a radioactive isotope after t years is given by $W_t = 2500 \times 3^{-\frac{t}{3000}}$ grams.

a Find the initial weight of the isotope.

b Find the time taken for the isotope to reduce to 30% of its original weight.

c Find the percentage weight loss after 1500 years.

d Sketch the graph of W_t against t.

6 Show that the solution to $16^x - 5 \times 8^x = 0$ is $x = \log_2 5$.

7 Solve for x, giving exact answers:

a $\ln x = 5$ **b** $3\ln x + 2 = 0$ **c** $e^x = 400$

d $e^{2x+1} = 11$ **e** $25e^{\frac{x}{2}} = 750$

8 A population of seals is given by $P_t = P_0 2^{\frac{t}{3}}$ where t is the time in years, $t \geqslant 0$.

a Find the time required for the population to double in size.

b Find the percentage increase in population during the first 4 years.

9 Consider $g : x \mapsto 2e^x - 5$.

a Find the defining equation of g^{-1}.

b Sketch the graphs of g and g^{-1} on the same set of axes.

c State the domain and range of g and g^{-1}.

d State the asymptotes and intercepts of g and g^{-1}.

10 Consider $f(x) = e^x$ and $g(x) = \ln(x+4)$, $x > -4$. Find:

a $(f \circ g)(5)$ **b** $(g \circ f)(0)$

11 **a** Sketch the graph of $f(x) = x^3 + x^2 - 6x - e^x$.

b Hence find all $x \in \mathbb{R}$ for which $e^x < x^3 + x^2 - 6x$.

Section D, Part 3

1 Without using a calculator, find the base 10 logarithms of:

a $\sqrt{1000}$ **b** $\frac{10}{\sqrt[3]{10}}$ **c** $\frac{10^a}{10^{-b}}$

2 Simplify:

a $e^{4\ln x}$ **b** $\ln(e^5)$ **c** $\ln(\sqrt{e})$

d $10^{\log x + \log 3}$ **e** $\ln\left(\frac{1}{e^x}\right)$ **f** $\frac{\log x^2}{\log_3 9}$

3 Write in the form e^x, where x is correct to 4 decimal places:

a 20 **b** 3000 **c** 0.075

4 Solve for x:

a $\log x = 3$ **b** $\log_3(x+2) = 1.732$ **c** $\log_2\left(\frac{x}{10}\right) = -0.671$

5 Write as a single logarithm:

a $\ln 60 - \ln 20$ **b** $\ln 4 + \ln 1$ **c** $\ln 200 - \ln 8 + \ln 5$

6 Write as logarithmic equations:

a $M = ab^n$ **b** $T = \frac{5}{\sqrt{l}}$ **c** $G = \frac{a^2 b}{c}$

7 Solve for x:

a $3^x = 300$ **b** $30 \times 5^{1-x} = 0.15$ **c** $3^{x+2} = 2^{1-x}$

8 Solve exactly for x:

a $e^{2x} = 3e^x$ **b** $e^{2x} - 7e^x + 12 = 0$

9 Write the following equations without logarithms:

a $\ln P = 1.5 \ln Q + \ln T$ **b** $\ln M = 1.2 - 0.5 \ln N$

10 Consider the functions $f(x) = e^{x^2} - x^6$ and $g(x) = \ln(x^2+1)$.

a Explain why $f(x)$ and $g(x)$ are even functions.

b Graph $y = f(x)$ and $y = g(x)$ on the domain $0 \leqslant x \leqslant 5$. Find the points of intersection of the graphs on this domain.

c Hence solve $x^6 - e^{x^2} + \ln(x^2+1) > 0$ for all $x \in \mathbb{R}$.

11 For the function $g : x \mapsto \log_3(x+2) - 2$:

a Find the domain and range.

b Find any asymptotes and axes intercepts for the graph of the function.

c Find the defining equation for g^{-1}. Explain how to verify your answer.

d Sketch the graphs of g, g^{-1}, and $y = x$ on the same axes.

12 The weight of a radioactive isotope remaining after t weeks is given by $W_t = 8000 \times e^{-\frac{t}{20}}$ grams. Find the time for the weight to:

a halve **b** reach 1000 g **c** reach 0.1% of its original value.

Section E, Part 1 (No Calculator)

1 If $f(x) = x^2 - 2x$, find in simplest form:

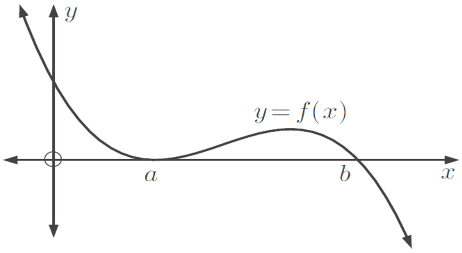
a $f(3)$ **b** $f(2x)$ **c** $f(-x)$ **d** $3f(x) - 2$

2 If $f(x) = 5 - x - x^2$, find in simplest form:

a $f(-1)$ **b** $f(x-1)$ **c** $f\left(\frac{x}{2}\right)$ **d** $2f(x) - f(-x)$

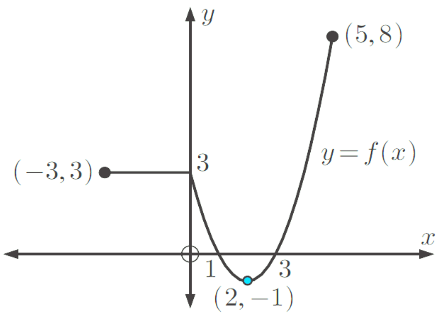
3 The graph of $f(x) = 3x^3 - 2x^2 + x + 2$ is translated to its image $g(x)$ by the vector $\binom{1}{-2}$. Write the equation of $g(x)$ in the form $g(x) = ax^3 + bx^2 + cx + d$.

4 The graph of $y = f(x)$ is shown alongside.
The x-axis is a tangent to $f(x)$ at $x = a$ and $f(x)$ cuts the x-axis at $x = b$.
On the same diagram, sketch the graph of $y = f(x - c)$ where $0 < c < b - a$.
Indicate the x-intercepts of $y = f(x - c)$.



5 For the graph of $y = f(x)$ given, sketch graphs of:

a $y = f(-x)$ **b** $y = -f(x)$
c $y = f(x + 2)$ **d** $y = f(x) + 2$



6 Consider the function $f : x \mapsto x^2$.
On the same set of axes graph:

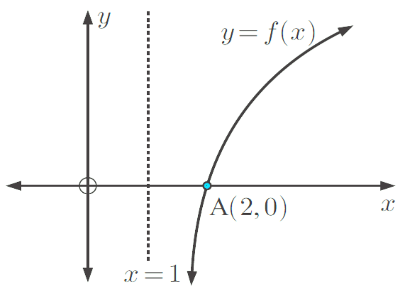
a $y = f(x)$ **b** $y = f(x - 1)$ **c** $y = 3f(x - 1)$ **d** $y = 3f(x - 1) + 2$

7 The graph of $y = f(x)$ is shown alongside.

a Sketch the graph of $y = g(x)$ where $g(x) = f(x + 3) - 1$.

b State the equation of the vertical asymptote of $y = g(x)$.

c Identify the point A′ on the graph of $y = g(x)$ which corresponds to point A.



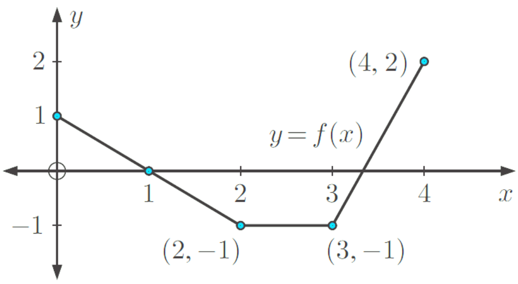
8 The graph of $y = f(x)$ is drawn alongside.

a Draw the graphs of $y = f(x)$ and $y = |f(x)|$ on the same set of axes.

b Find the y-intercept of $\dfrac{1}{f(x)}$.

c Show on the diagram the points that are invariant for the function $\dfrac{1}{f(x)}$.

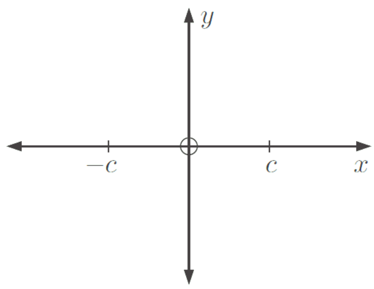
d Draw the graphs of $y = f(x)$ and $y = \dfrac{1}{f(x)}$ on the same set of axes.



9 Let $f(x) = \dfrac{c}{x + c}$, $x \neq -c$, $c > 0$.

a On a set of axes like those shown, sketch the graph of $y = f(x)$. Label clearly any points of intersection with the axes and any asymptotes.

b On the same set of axes, sketch the graph of $y = \dfrac{1}{f(x)}$.
Label clearly any points of intersection with the axes.



Section E, Part 2 (Calculator)

1 Use your calculator to help graph $f(x) = (x+1)^2 - 4$. Include all axes intercepts, and the coordinates of the turning point of the function.

2 Consider the function $f : x \mapsto x^2$. On the same set of axes graph:

a $y = f(x)$ **b** $y = f(x+2)$ **c** $y = 2f(x+2)$ **d** $y = 2f(x+2) - 3$

3 Consider $f : x \mapsto \dfrac{2^x}{x}$.

a Does the function have any axes intercepts?

b Find the equations of the asymptotes of the function.

c Find any turning points of the function.

d Sketch the function for $-4 \leqslant x \leqslant 4$.

4 Consider $f(x) = 2^{-x}$.

a Use your calculator to help determine whether the following are true or false:

i As $x \to \infty$, $2^{-x} \to 0$. **ii** As $x \to -\infty$, $2^{-x} \to 0$.

iii The y-intercept is $\frac{1}{2}$. **iv** $2^{-x} > 0$ for all x.

b On the same set of axes, graph $y = f(x)$ and $y = |f(x)|$.

c Write down the equation of any asympotes of $y = |f(x)|$.

5 The graph of the function $f(x) = (x+1)^2 + 4$ is translated 2 units to the right and 4 units up.

a Find the function $g(x)$ corresponding to the translated graph.

b State the range of $f(x)$.

c State the range of $g(x)$.

6 For each of the following functions:

i Find $y = f(x)$, the result when the function is translated by $\binom{1}{-2}$.

ii Sketch the original function and its translated function on the same set of axes. Clearly state any asymptotes of each function.

iii State the domain and range of each function.

a $y = \dfrac{1}{x}$ **b** $y = 2^x$

7 Sketch the graph of $f(x) = x^2 + 1$, and on the same set of axes sketch the graphs of:

a $-f(x)$ **b** $f(2x)$ **c** $f(x) + 3$

8 Suppose $f(x) = x + 2$. The function F is obtained by stretching the function f vertically with scale factor 2, then stretching it horizontally with scale factor $\frac{1}{2}$, then translating it $\frac{1}{2}$ horizontally and -3 vertically.

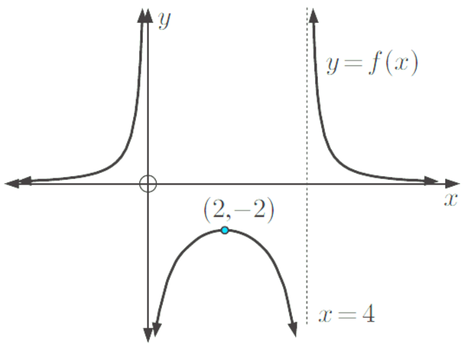
a Find the function $F(x)$.

b What can be said about the point $(1, 3)$ under this transformation?

c What happens to the points $(0, 2)$ and $(-1, 1)$ under this transformation?

d Show that the points in **c** also lie on the graph of $y = F(x)$.

9 The graph of $y = f(x)$ is given.
On the same set of axes graph each pair of functions:



a $y = f(x)$ and $y = f(x-2)+1$

b $y = f(x)$ and $y = \frac{1}{f(x)}$

c $y = f(x)$ and $y = |f(x)|$

10 Consider the function $f(x) = \frac{2x-3}{3x+5}$.

a Find the asymptotes of $y = f(x)$.

b Discuss the behaviour of the graph near these asymptotes.

c Find the axes intercepts of $y = f(x)$.

d Sketch the graph of $y = f(x)$.

e Describe the transformations which transform $y = \frac{1}{x}$ into $y = f(x)$.

f Describe the transformations which transform $y = f(x)$ into $y = \frac{1}{x}$.

11 **a** Sketch the graph of $f(x) = -2x + 3$, clearly showing the axes intercepts.

b Find the invariant points for the graph of $y = \frac{1}{f(x)}$.

c State the equation of the vertical asymptote of $y = \frac{1}{f(x)}$ and find its y-intercept.

d Sketch the graph of $y = \frac{1}{f(x)}$ on the same axes as in part **a**, showing clearly the information you have found.

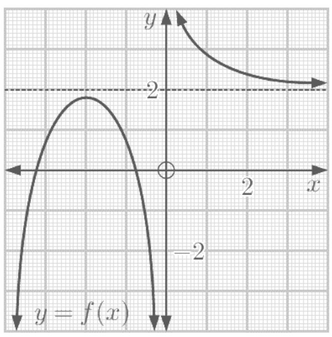
e On a new pair of axes, sketch the graphs of $y = |f(x)|$ and $y = f(|x|)$ showing clearly all important features.

Section E, Part 3 (Calculator)

1 If $f(x) = \frac{4}{x}$, find in simplest form:

a $f(-4)$ **b** $f(2x)$ **c** $f\left(\frac{x}{2}\right)$ **d** $4f(x+2)-3$

2 Consider the graph of $y = f(x)$ shown.



a Use the graph to determine:

i the coordinates of the turning point

ii the equation of the vertical asymptote

iii the equation of the horizontal asymptote

iv the x-intercepts.

b Graph the function $g : x \mapsto x+1$ on the same set of axes.

c Hence estimate the coordinates of the points of intersection of $y = f(x)$ and $y = g(x)$.

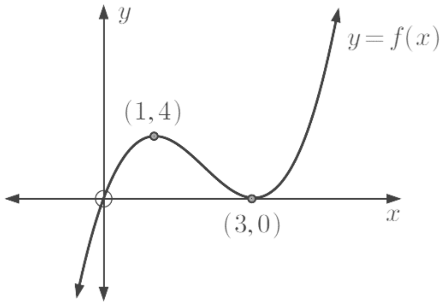
3 Sketch the graph of $f(x) = -x^2$, and on the same set of axes sketch the graph of:

a $y = f(-x)$ **b** $y = -f(x)$ **c** $y = f(2x)$ **d** $y = f(x-2)$

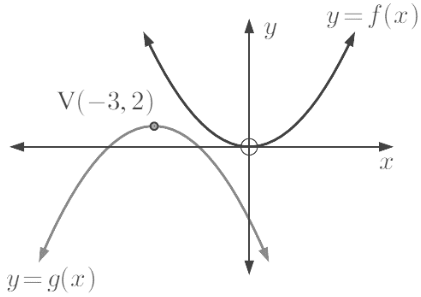
4 The graph of a cubic function $y = f(x)$ is shown alongside.

a Sketch the graph of $g(x) = -f(x-1)$.

b State the coordinates of the turning points of $y = g(x)$.



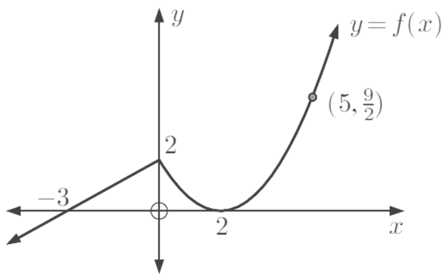
5 The graph of $f(x) = x^2$ is transformed to the graph of $g(x)$ by a reflection and a translation as illustrated. Find the formula for $g(x)$ in the form $g(x) = ax^2 + bx + c$.



6 Given the graph of $y = f(x)$, sketch graphs of:

a $f(-x)$ **b** $f(x+1)$

c $f(x) - 3$



7 The graph of $f(x) = x^3 + 3x^2 - x + 4$ is translated to its image $y = g(x)$ by the vector $\begin{pmatrix} -1 \\ 3 \end{pmatrix}$. Write the equation of $g(x)$ in the form $g(x) = ax^3 + bx^2 + cx + d$.

8 **a** Find the equation of the line that results when the line $f(x) = 3x + 2$ is translated:

i 2 units to the left **ii** 6 units upwards.

b Show that when the linear function $f(x) = ax + b$, $a > 0$ is translated k units to the left, the resulting line is the same as when $f(x)$ is translated ka units upwards.

9 The function $f(x)$ results from transforming the function $y = \frac{1}{x}$ by a reflection in the y-axis, then a vertical stretch with scale factor 3, then a translation of $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

a Find an expression for $f(x)$.

b Sketch $y = f(x)$ and state its domain and range.

c Does $y = f(x)$ have an inverse function? Explain your answer.

d Is the function f a self-inverse function? Give graphical and algebraic evidence to support your answer.

10 Consider $y = \log_4 x$.

- **a** Find the function which results from a translation of $\binom{1}{-2}$.
- **b** Sketch the original function and the translated function on the same set of axes.
- **c** State the asymptotes of each function.
- **d** State the domain and range of each function.

11 The function $g(x)$ results when $y = \frac{1}{x}$ is transformed by a vertical stretch with scale factor $\frac{1}{3}$, followed by a reflection in the y-axis, followed by a translation of 2 units to the right.

- **a** Write an expression for $g(x)$ in the form $g(x) = \frac{ax+b}{cx+d}$.
- **b** Find the asymptotes of $y = g(x)$.
- **c** State the domain and range of $g(x)$.
- **d** Sketch $y = g(x)$.

Section F, Part 1 (No Calculator)

1 Identify the following sequences as arithmetic, geometric, or neither:

- **a** $7, -1, -9, -17, ...$ **b** $9, 9, 9, 9, ...$ **c** $4, -2, 1, -\frac{1}{2}, ...$
- **d** $1, 1, 2, 3, 5, 8, ...$ **e** the set of all multiples of 4 in ascending order.

2 Find k if $3k$, $k-2$, and $k+7$ are consecutive terms of an arithmetic sequence.

3 Show that $28, 23, 18, 13, ...$ is an arithmetic sequence. Hence find u_n and the sum S_n of the first n terms in simplest form.

4 Find k given that 4, k, and $k^2 - 1$ are consecutive terms of a geometric sequence.

5 Determine the general term of a geometric sequence given that its sixth term is $\frac{16}{3}$ and its tenth term is $\frac{256}{3}$.

6 Insert six numbers between 23 and 9 so that all eight numbers are in arithmetic sequence.

7 Find, in simplest form, a formula for the general term u_n of:

- **a** $86, 83, 80, 77, ...$ **b** $\frac{3}{4}, 1, \frac{7}{6}, \frac{9}{7}, ...$ **c** $100, 90, 81, 72.9, ...$

Hint: One of these sequences is neither arithmetic nor geometric.

8 Expand and hence evaluate: **a** $\sum_{k=1}^{7} k^2$ **b** $\sum_{k=1}^{4} \frac{k+3}{k+2}$

9 Find the sum of each of the following infinite geometric series:

- **a** $18 - 12 + 8 - ....$ **b** $8 + 4\sqrt{2} + 4 + ...$

10 A ball bounces from a height of 3 metres and returns to 80% of its previous height on each bounce. Find the total distance travelled by the ball until it stops bouncing.

11 The sum of the first n terms of an infinite sequence is $\frac{3n^2+5n}{2}$ for all $n \in \mathbb{Z}^+$.

- **a** Find the nth term. **b** Prove that the sequence is arithmetic.

12 a, b, and c are consecutive terms of both an arithmetic and geometric sequence.
What can be deduced about a, b, and c?

13 x, y, and z are consecutive terms of a geometric sequence.
If $x + y + z = \frac{7}{3}$ and $x^2 + y^2 + z^2 = \frac{91}{9}$, find the values of x, y, and z.

14 $2x$ and $x-2$ are the first two terms of a convergent series. The sum of the series is $\frac{18}{7}$. Find x, clearly explaining why there is only one possible value.

15 a, b, and c are consecutive terms of an arithmetic sequence. Prove that the following are also consecutive terms of an arithmetic sequence:

a $b+c,\quad c+a,\quad$ and $\quad a+b$ **b** $\dfrac{1}{\sqrt{b}+\sqrt{c}},\quad \dfrac{1}{\sqrt{c}+\sqrt{a}},\quad$ and $\quad \dfrac{1}{\sqrt{a}+\sqrt{b}}$

Section F , Part 2 (Calculator)

1 List the first four members of the sequences defined by:

a $u_n = 3^{n-2}$ **b** $u_n = \dfrac{3n+2}{n+3}$ **c** $u_n = 2^n - (-3)^n$

2 A sequence is defined by $u_n = 6(\frac{1}{2})^{n-1}$.

a Prove that the sequence is geometric. **b** Find u_1 and r.

c Find the 16th term of the sequence to 3 significant figures.

3 Consider the sequence $24, 23\frac{1}{4}, 22\frac{1}{2}, ...$

a Which term of the sequence is -36? **b** Find the value of u_{35}.

c Find S_{40}, the sum of the first 40 terms of the sequence.

4 Find the sum of:

a the first 23 terms of $3+9+15+21+....$

b the first 12 terms of $24+12+6+3+....$

5 Find the first term of the sequence $5, 10, 20, 40, ...$ which exceeds 10 000.

6 What will an investment of €6000 at 7% p.a. compound interest amount to after 5 years if the interest is compounded:

a annually **b** quarterly **c** monthly?

7 The nth term of a sequence is given by the formula $u_n = 5n - 8$.

a Find the value of u_{10}.

b Write down an expression for $u_{n+1} - u_n$ and simplify it.

c Hence explain why the sequence is arithmetic.

d Evaluate $u_{15} + u_{16} + u_{17} + ... + u_{30}$.

8 A geometric sequence has $u_6 = 24$ and $u_{11} = 768$. Determine the general term of the sequence and hence find:

a u_{17} **b** the sum of the first 15 terms.

9 Find the first term of the sequence $24, 8, \frac{8}{3}, \frac{8}{9}, ...$ which is less than 0.001 .

10 **a** Determine the number of terms in the sequence $128, 64, 32, 16, ..., \frac{1}{512}$.

b Find the sum of these terms.

11 Find the sum of each of the following infinite geometric series:

a $1.21 - 1.1 + 1 - ....$ **b** $\frac{14}{3} + \frac{4}{3} + \frac{8}{21} + ...$

12 How much should be invested at a fixed rate of 9% p.a. compound interest if you need it to amount to $20 000 after 4 years with interest paid monthly?

13 In 2004 there were 3000 iguanas on a Galapagos island. Since then, the population of iguanas on the island has increased by 5% each year.

- **a** How many iguanas were on the island in 2007?
- **b** In what year will the population first exceed 10 000?

Section F , Part 3 (Calculator)

1 A sequence is defined by $u_n = 68 - 5n$.

- **a** Prove that the sequence is arithmetic.
- **b** Find u_1 and d.
- **c** Find the 37th term of the sequence.
- **d** State the first term of the sequence which is less than -200.

2
- **a** Show that the sequence $3, 12, 48, 192, ...$ is geometric.
- **b** Find u_n and hence find u_9.

3 Find the general term of the arithmetic sequence with $u_7 = 31$ and $u_{15} = -17$. Hence, find the value of u_{34}.

4 Write using sigma notation:

- **a** $4 + 11 + 18 + 25 + ...$ for n terms
- **b** $\frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \frac{1}{32} + ...$ for n terms.

5 Evaluate:
- **a** $\sum_{k=1}^{8} \left(\frac{31-3k}{2}\right)$
- **b** $\sum_{k=1}^{15} 50(0.8)^{k-1}$
- **c** $\sum_{k=7}^{\infty} 5\left(\frac{2}{5}\right)^{k-1}$

6 How many terms of the series $11 + 16 + 21 + 26 + ...$ are needed to exceed a sum of 450?

7 £12 500 is invested in an account which pays 8.25% p.a. compounded. Find the value of the investment after 5 years if the interest is compounded:

- **a** half-yearly
- **b** monthly.

8 The sum of the first two terms of an infinite geometric series is 90. The third term is 24.

- **a** Show that there are two possible series. Find the first term and common ratio in each case.
- **b** Show that both series converge and find their respective sums.

9 Seve is training for a long distance walk. He walks 10 km in the first week, then each week thereafter he walks an additional 500 m. If he continues this pattern for a year, how far does Seve walk:

- **a** in the last week
- **b** in total?

10
- **a** Under what conditions will the series $\sum_{k=1}^{\infty} 50(2x-1)^{k-1}$ converge? Explain your answer.
- **b** Find $\sum_{k=1}^{\infty} 50(2x-1)^{k-1}$ if $x = 0.3$.

11 a, b, c, d, and e are consecutive terms of an arithmetic sequence.
Prove that $a + e = b + d = 2c$.

12 Suppose n consecutive geometric terms are inserted between 1 and 2. Write the sum of these n terms, in terms of n.

13 An arithmetic sequence, and a geometric sequence with common ratio r, have the same first two terms. Show that the third term of the geometric sequence is $\frac{r^2}{2r-1}$ times the third term of the arithmetic sequence.

14 $11 - 2 = 9 = 3^2$ and $1111 - 22 = 1089 = 33^2$.
Show that $\underbrace{(111111....1)}_{2n \text{ 1s}} - \underbrace{(22222....2)}_{n \text{ 2s}}$ is a perfect square.

Section G, Part 1 (No Calculator)

1 Convert these to radians in terms of π:

a $120°$ **b** $225°$ **c** $150°$ **d** $540°$

2 Find the acute angles that would have the same:

a sine as $\frac{2\pi}{3}$ **b** sine as $165°$ **c** cosine as $276°$.

3 Find:

a $\sin 159°$ if $\sin 21° \approx 0.358$ **b** $\cos 92°$ if $\cos 88° \approx 0.035$

c $\cos 75°$ if $\cos 105° \approx -0.259$ **d** $\sin(-133°)$ if $\sin 47° \approx 0.731$.

4 Use a unit circle diagram to find:

a $\cos 360°$ and $\sin 360°$ **b** $\cos(-\pi)$ and $\sin(-\pi)$.

5 Explain how to use the unit circle to find θ when $\cos\theta = -\sin\theta$, $0 \leqslant \theta \leqslant 2\pi$.

6 Find exact values for $\sin\theta$, $\cos\theta$, and $\tan\theta$ for θ equal to:

a $\frac{2\pi}{3}$ **b** $\frac{8\pi}{3}$

7 If $\sin x = -\frac{1}{4}$ and $\pi < x < \frac{3\pi}{2}$, find $\tan x$ exactly.

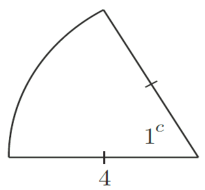
8 If $\cos\theta = \frac{3}{4}$ find the possible values of $\sin\theta$.

9 Evaluate:

a $2\sin(\frac{\pi}{3})\cos(\frac{\pi}{3})$ **b** $\tan^2(\frac{\pi}{4}) - 1$ **c** $\cos^2(\frac{\pi}{6}) - \sin^2(\frac{\pi}{6})$

10 Given $\tan x = -\frac{3}{2}$ and $\frac{3\pi}{2} < x < 2\pi$, find: **a** $\sin x$ **b** $\cos x$.

11



Find the perimeter and area of the sector.

12 Suppose $\cos\theta = \frac{\sqrt{11}}{\sqrt{17}}$ and θ is acute. Find the exact value of $\tan\theta$.

13 Simplify:

a $\cos\left(\frac{\pi}{2} - \theta\right) - \sin\theta$ **b** $\cos(-\theta)\tan\theta$ **c** $\sin(-\alpha)\cos\left(\alpha - \frac{\pi}{2}\right)$

Section G, Part 2 (Calculator)

1 Determine the coordinates of the point on the unit circle corresponding to an angle of:

a $320°$ **b** $163°$

2 Convert to radians to 4 significant figures:

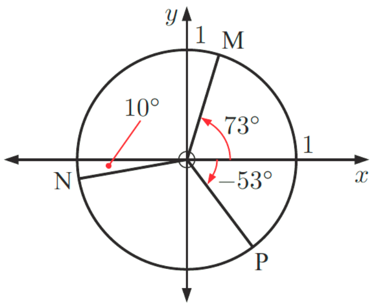
a $71°$ **b** $124.6°$ **c** $-142°$

3 Convert these radian measurements to degrees, to 2 decimal places:

a 3 **b** 1.46 **c** 0.435 **d** -5.271

4 Determine the area of a sector of angle $\frac{5\pi}{12}$ and radius 13 cm.

5 Find the coordinates of the points M, N, and P on the unit circle.



6 Find the angle [OA] makes with the positive x-axis if the x-coordinate of the point A on the unit circle is -0.222.

7 Find all angles between $0°$ and $360°$ which have:

a a cosine of $-\frac{\sqrt{3}}{2}$ **b** a sine of $\frac{1}{\sqrt{2}}$ **c** a tangent of $-\sqrt{3}$

8 Find θ for $0 \leqslant \theta \leqslant 2\pi$ if:

a $\cos\theta = -1$ **b** $\sin^2\theta = \frac{3}{4}$

9 Find the obtuse angles which have the same:

a sine as $47°$ **b** sine as $\frac{\pi}{15}$ **c** cosine as $186°$

10 Find the perimeter and area of a sector of radius 11 cm and angle $63°$.

11 Find the radius and area of a sector of perimeter 36 cm with an angle of $\frac{2\pi}{3}$.

12 Find two angles on the unit circle with $0 \leqslant \theta \leqslant 2\pi$, such that:

a $\cos\theta = \frac{2}{3}$ **b** $\sin\theta = -\frac{1}{4}$ **c** $\tan\theta = 3$

Section G, Part 3 (Calculator)

1 Convert these radian measurements to degrees:

a $\frac{2\pi}{5}$ **b** $\frac{5\pi}{4}$ **c** $\frac{7\pi}{9}$ **d** $\frac{11\pi}{6}$

2 Illustrate the regions where $\sin\theta$ and $\cos\theta$ have the same sign.

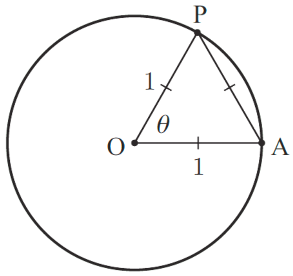
3 Use a unit circle diagram to find:

a $\cos(\frac{3\pi}{2})$ and $\sin(\frac{3\pi}{2})$ **b** $\cos(-\frac{\pi}{2})$ and $\sin(-\frac{\pi}{2})$

4 Suppose $m = \sin p$, where p is acute. Write an expression in terms of m for:

a $\sin(\pi - p)$ **b** $\sin(p + 2\pi)$ **c** $\cos p$ **d** $\tan p$

5



a State the value of θ in:

i degrees **ii** radians.

b State the arc length AP.

c State the area of the minor *sector* OAP.

6 Without a calculator, evaluate $\tan^2(\frac{2\pi}{3})$.

7 Show that $\cos(\frac{3\pi}{4}) - \sin(\frac{3\pi}{4}) = -\sqrt{2}$.

8 If $\cos\theta = -\frac{3}{4}$, $\frac{\pi}{2} < \theta < \pi$ find the exact value of:

a $\sin\theta$ **b** $\tan\theta$ **c** $\sin(\theta + \pi)$

9 Without using a calculator, evaluate:

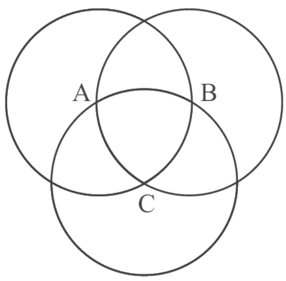
a $\tan^2 60° - \sin^2 45°$ **b** $\cos^2(\frac{\pi}{4}) + \sin(\frac{\pi}{2})$ **c** $\cos(\frac{5\pi}{3}) - \tan(\frac{5\pi}{4})$

10 Simplify:

a $\sin(\pi - \theta) - \sin\theta$ **b** $\sin\left(\frac{\pi}{2} - \theta\right) - 2\cos\theta$

11 Use a unit circle diagram to show that $\cos\left(\frac{\pi}{2} + \theta\right) = -\sin\theta$ for $\frac{\pi}{2} < \theta < \pi$.

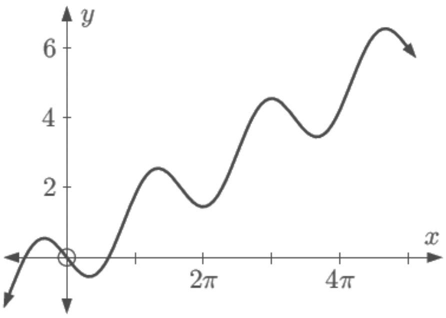
12 Three circles with radius r are drawn as shown, each with its centre on the circumference of the other two circles. A, B and C are the centres of the three circles. Prove that an expression for the area of the shaded region is $A = \frac{r^2}{2}(\pi - \sqrt{3})$.



Section H, Part 1 (No Calculator)

1 Which of the following graphs display periodic behaviour?

a



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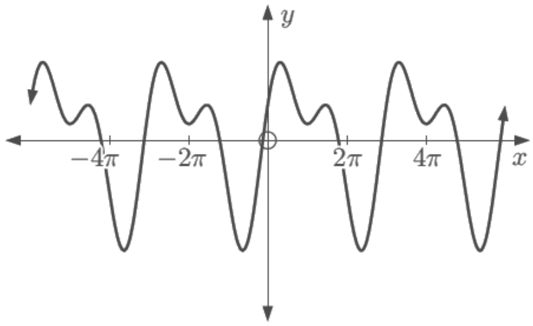


2 Draw the graph of $y = 4\sin x$ for $0 \leqslant x \leqslant 2\pi$.

3 State the minimum and maximum values of:

a $1 + \sin x$ **b** $-2\cos 3x$

4 State the period of:

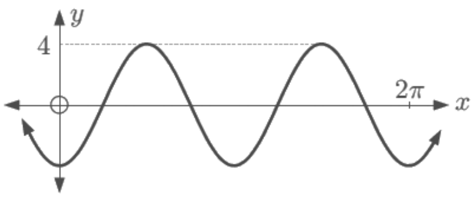
a $y = 4\sin(\frac{x}{5})$ **b** $y = -2\cos(4x)$ **c** $y = 4\cos(\frac{x}{2}) + 4$ **d** $y = \frac{1}{2}\tan(3x)$

5 Complete the table:

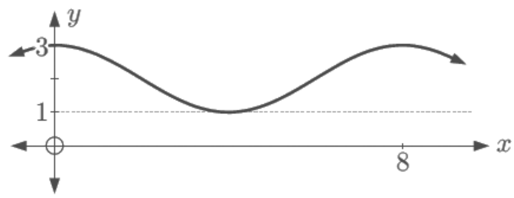
Function	*Period*	*Amplitude*	*Domain*	*Range*
$y = -3\sin(\frac{x}{4}) + 1$				
$y = \tan 2x$				
$y = 3\cos \pi x$				

6 Find the cosine function represented in each of the following graphs:

a



b



7 State the transformations which map:

a $y = \sin x$ onto $y = 3\sin(2x)$ **b** $y = \cos x$ onto $y = \cos\left(x - \frac{\pi}{3}\right) - 1$

8 Find the remaining five trigonometric ratios from sin, cos, tan, csc, sec, and cot, if:

a $\cos x = \frac{1}{3}$ and $0 < x < \pi$ **b** $\tan x = \frac{4}{5}$ and $\pi < x < 2\pi$.

9 Simplify:

a $\arctan(\tan(-0.5))$ **b** $\arcsin(\sin(-\frac{\pi}{6}))$ **c** $\arccos(\cos 2\pi)$

Section H, Part 2 (Calculator)

1 For each set of data below, draw a scatter diagram and state if the data exhibits approximately periodic behaviour.

a

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	2.7	0.8	−1.7	−3	−2.1	0.3	2.5	2.9	1.3	−1.3	−2.9	−2.5	−0.3

b

x	0	1	2	3	4	5	6	7	8	9
y	5	3.5	6	−1.5	4	−2.5	−0.8	0.9	2.6	4.3

2 Draw the graph of $y = \sin 3x$ for $0 \leqslant x \leqslant 2\pi$.

3 State the period of: **a** $y = 4\sin(\frac{x}{3})$ **b** $y = -2\tan 4x$

4 Draw the graph of $y = 0.6\cos(2.3x)$ for $0 \leqslant x \leqslant 5$.

5 A robot on Mars records the temperature every Mars day. A summary series, showing every one hundredth Mars day, is shown in the table below.

Number of Mars days	0	100	200	300	400	500	600	700	800	900	1000	1100	1200	1300
Temp. (°C)	−43	−15	−5	−21	−59	−79	−68	−50	−27	−8	−15	−70	−78	−68

a Find the maximum and minimum temperatures recorded by the robot.

b Find a sine model for the temperature T in terms of the number of Mars days n.

c Use this information to estimate the length of a Mars year.

6 State the minimum and maximum values of:

a $y = 5\sin x - 3$ **b** $y = \frac{1}{3}\cos x + 1$

7 State the transformations which map:

a $y = \tan x$ onto $y = -\tan(2x)$ **b** $y = \sin x$ onto $y = 2\sin\left(\frac{x}{2} - \frac{\pi}{4}\right) + \frac{1}{2}$

8 **a** Sketch the graphs of $y = \sec x$ and $y = \csc x$ on the same set of axes for $-2\pi \leqslant x \leqslant 2\pi$.

b State a transformation which maps $y = \sec x$ onto $y = \csc x$ for all $x \in \mathbb{R}$.

9 **a** Sketch the graphs of $y = \arcsin x$ and $y = \arccos x$ on the same set of axes.

b State the domain and range of each function.

c State the transformations which map $y = \arcsin x$ onto $y = \arccos x$.

Section H , Part 3 (Calculator)

1 Find b given that the function $y = \sin bx$, $b > 0$ has period:

a 6π **b** $\frac{\pi}{12}$ **c** 9

2 **a** Without using technology, draw the graph of $f(x) = \sin(x - \frac{\pi}{3}) + 2$ for $0 \leqslant x \leqslant 2\pi$.

b For what values of k will $f(x) = k$ have solutions?

3 Consider the graph alongside.

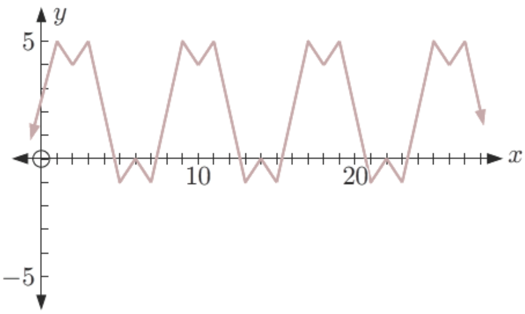
a Explain why this graph shows periodic behaviour.

b State:

i the period

ii the maximum value

iii the minimum value



4 On the same set of axes, for the domain $0 \leqslant x \leqslant 2\pi$, sketch:

a $y = \cos x$ and $y = \cos x - 3$

b $y = \cos x$ and $y = \cos(x - \frac{\pi}{4})$

c $y = \cos x$ and $y = 3\cos 2x$

d $y = \cos x$ and $y = 2\cos(x - \frac{\pi}{3}) + 3$

5 The table below gives the mean monthly maximum temperature for Perth Airport in Western Australia.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Temp. (°C)	31.5	31.8	29.5	25.4	21.5	18.8	17.7	18.3	20.1	22.4	25.5	28.8

a A sine function of the form $T \approx a\sin(b(t - c)) + d$ is used to model the data. Find good estimates of the constants a, b, c, and d without using technology. Use Jan $\equiv 1$, Feb $\equiv 2$, and so on.

b Check your answer to **a** using technology. How well does your model fit?

6 State the transformations which map:

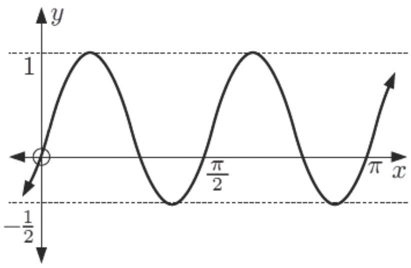
a $y = \cos x$ onto $y = \cos(x - \frac{\pi}{3}) + 1$

b $y = \tan x$ onto $y = -2\tan x$

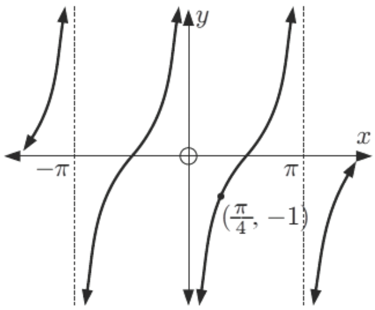
c $y = \sin x$ onto $y = \sin(3x)$

7 Find the function represented in each of the following graphs:

a



b



8 Simplify:

a $\csc x \tan x$

b $\dfrac{\tan x}{\sec x}$

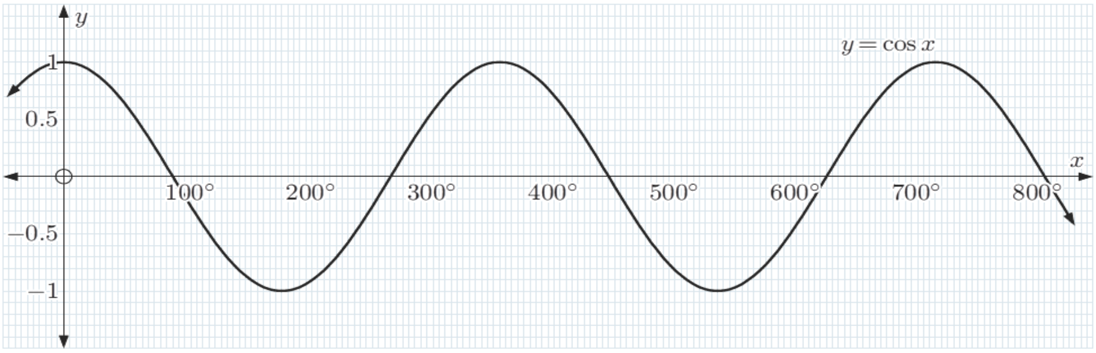
c $\sec x - \tan x \sin x$

9 **a** For what restricted domain of $y = \tan x$, is $y = \arctan x$ the inverse function?

b Sketch $y = \tan x$ for this domain, and $y = \arctan x$, on the same set of axes.

Section I , Part 1 (No Calculator)

1



Use the graph of $y = \cos x$ to find the solutions of:

a $\cos x = -0.4, \quad 0 \leqslant x \leqslant 800°$ **b** $\cos x = 0.9, \quad 0 \leqslant x \leqslant 600°$

2 Solve in terms of π:

a $2\sin x = -1$ for $0 \leqslant x \leqslant 4\pi$ **b** $\sqrt{2}\sin x - 1 = 0$ for $-2\pi \leqslant x \leqslant 2\pi$

3 Find the x-intercepts of:

a $y = 2\sin 3x + \sqrt{3}$ for $0 \leqslant x \leqslant 2\pi$ **b** $y = \sqrt{2}\sin\left(x + \frac{\pi}{4}\right)$ for $0 \leqslant x \leqslant 3\pi$

4 Solve algebraically in terms of π:

a $\cot x = \sqrt{3}$ for $x \in [0, 2\pi]$ **b** $\sec^2 x = \tan x + 1$ for $x \in [0, 2\pi]$

5 Simplify: **a** $\cos\left(\frac{3\pi}{2} - \theta\right)$ **b** $\sin\left(\theta + \frac{\pi}{2}\right)$

6 Simplify: **a** $\dfrac{1 - \cos^2\theta}{1 + \cos\theta}$ **b** $\dfrac{\sin\alpha - \cos\alpha}{\sin^2\alpha - \cos^2\alpha}$ **c** $\dfrac{4\sin^2\alpha - 4}{8\cos\alpha}$

7 If $\sin\alpha = -\frac{3}{4}$, $\pi \leqslant \alpha \leqslant \frac{3\pi}{2}$, find the exact value of:

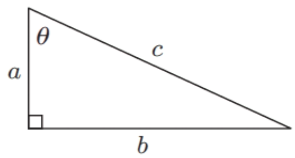
a $\cos\alpha$ **b** $\sin 2\alpha$ **c** $\cos 2\alpha$ **d** $\tan 2\alpha$

8 Show that $\dfrac{\sin 2\alpha - \sin\alpha}{\cos 2\alpha - \cos\alpha + 1}$ simplifies to $\tan\alpha$.

9 Find the exact value of: **a** $\cos(165°)$ **b** $\tan\left(\frac{\pi}{12}\right)$

10 Solve for x in $[0, 2\pi]$: **a** $2\cos 2x + 1 = 0$ **b** $\sin 2x = -\sqrt{3}\cos 2x$

11



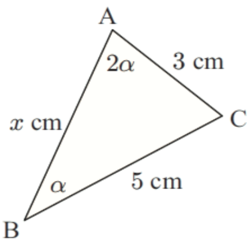
Prove that: **a** $\sin 2\theta = \dfrac{2ab}{c^2}$

b $\cos 2\theta = \dfrac{a^2 - b^2}{c^2}$

12 Find the exact solutions of:

a $\sqrt{2}\cos\left(x + \frac{\pi}{4}\right) - 1 = 0, \quad x \in [0, 4\pi]$ **b** $\tan 2x - \sqrt{3} = 0, \quad x \in [0, 2\pi]$

13



a Show that $\cos\alpha = \frac{5}{6}$.

b Show that x is a solution of $3x^2 - 25x + 48 = 0$.

c Find x by solving the equation in **b**.

Section I , Part 2 (Calculator)

1 Solve for $0 \leqslant x \leqslant 8$: **a** $\sin x = 0.382$ **b** $\tan(\frac{x}{2}) = -0.458$

2 Solve:

a $\cos x = 0.4379$ for $0 \leqslant x \leqslant 10$ **b** $\cos(x - 2.4) = -0.6014$ for $0 \leqslant x \leqslant 6$

3 If $\sin A = \frac{5}{13}$ and $\cos A = \frac{12}{13}$, find: **a** $\sin 2A$ **b** $\cos 2A$ **c** $\tan 2A$

4 **a** Solve for $0 \leqslant x \leqslant 10$:

i $\tan x = 4$ **ii** $\tan(\frac{x}{4}) = 4$ **iii** $\tan(x - 1.5) = 4$

b Find exact solutions for x given $-\pi \leqslant x \leqslant \pi$:

i $\tan(x + \frac{\pi}{6}) = -\sqrt{3}$ **ii** $\tan 2x = -\sqrt{3}$ **iii** $\tan^2 x - 3 = 0$

c Solve $3\tan(x - 1.2) = -2$ for $0 \leqslant x \leqslant 10$.

5 Show that:

a $\sqrt{2}\cos\left(\theta + \frac{\pi}{4}\right) = \cos\theta - \sin\theta$

b $\cos\alpha\cos(\beta - \alpha) - \sin\alpha\sin(\beta - \alpha) = \cos\beta$

6 If $\cos x = -\frac{3}{4}$ and $\pi < x < \frac{3\pi}{2}$ find the exact value of $\sin\left(\frac{x}{2}\right)$.

7 Solve for $0 \leqslant x \leqslant 2\pi$:

a $\cos x = 0.3$ **b** $2\sin(3x) = \sqrt{2}$ **c** $43 + 8\sin x = 50.1$

8 An ecologist studying a species of water beetle estimates the population of a colony over an eight week period. If t is the number of weeks after the initial estimate is made, then the population in thousands can be modelled by $P(t) = 5 + 2\sin(\frac{\pi t}{3})$ where $0 \leqslant t \leqslant 8$.

a What was the initial population?

b What were the smallest and largest populations?

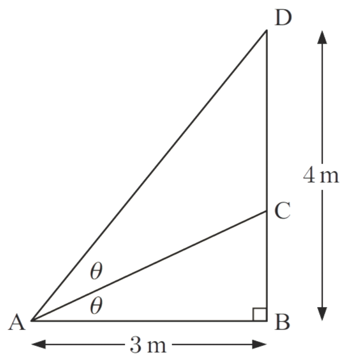
c During what time interval(s) did the population exceed 6000?

9 Solve for x: $3\cos x + \sin 2x = 1$ for $0 \leqslant x \leqslant 10$.

10 Write $3\sin x + 4\cos x$ in the form $k\cos(x + b)$, where $k > 0$ and $0 < b < 2\pi$.

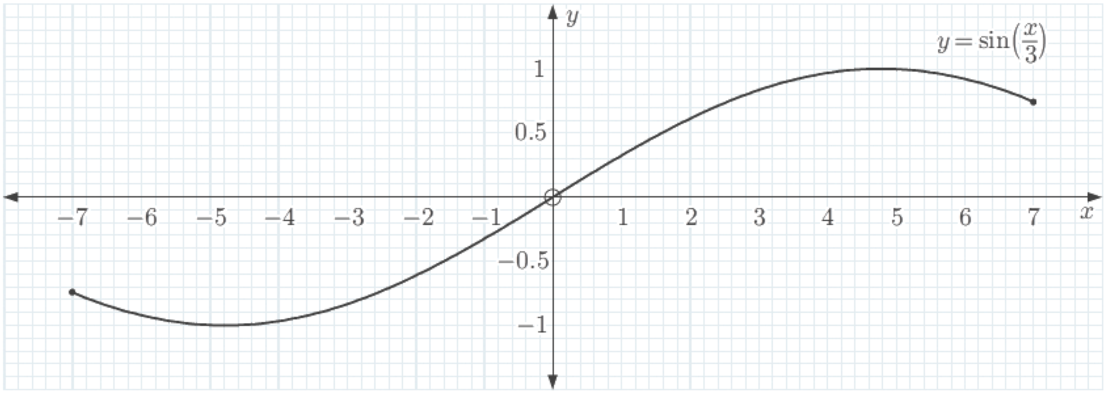
11 From ground level, a shooter is aiming at targets on a vertical brick wall. At the current angle of elevation of his rifle, he will hit a target 20 m above ground level. If he doubles the angle of elevation of the rifle, he will hit a target 45 m above ground level. How far is the shooter from the wall?

12 Find exactly the length of BC:



Section I , Part 3 (Calculator)

1 Consider $y = \sin(\frac{x}{3})$ on the domain $-7 \leqslant x \leqslant 7$. Use the graph to solve, correct to 1 decimal place: **a** $\sin(\frac{x}{3}) = -0.9$ **b** $\sin(\frac{x}{3}) = \frac{1}{4}$



2 Solve algebraically for $0 \leqslant x \leqslant 2\pi$, giving answers in terms of π:

a $\sin^2 x - \sin x - 2 = 0$ **b** $4\sin^2 x = 1$

3 Find the exact solutions of:

a $\tan(x - \frac{\pi}{3}) = \frac{1}{\sqrt{3}}$, $0 \leqslant x \leqslant 4\pi$ **b** $\cos(x + \frac{2\pi}{3}) = \frac{1}{2}$, $-2\pi \leqslant x \leqslant 2\pi$

4 Simplify:

a $\cos^3\theta + \sin^2\theta\cos\theta$ **b** $\dfrac{\cos^2\theta - 1}{\sin\theta}$ **c** $5 - 5\sin^2\theta$

d $\dfrac{\sin^2\theta - 1}{\cos\theta}$ **e** $\dfrac{\tan\theta + \cot\theta}{\sec\theta}$ **f** $\cos^2\theta(\tan\theta + 1)^2 - 1$

5 If $\tan 2\alpha = \frac{4}{3}$ for $\alpha \in \,]\,0, \frac{\pi}{2}\,[$, find the exact value of $\sin\alpha$ without using a calculator.

6 Simplify: **a** $(2\sin\alpha - 1)^2$ **b** $(\cos\alpha - \sin\alpha)^2$

7 Show that:

a $\dfrac{\cos\theta}{1 + \sin\theta} + \dfrac{1 + \sin\theta}{\cos\theta} = 2\sec\theta$ **b** $\left(1 + \dfrac{1}{\cos\theta}\right)(\cos\theta - \cos^2\theta) = \sin^2\theta$

8 Solve exactly: **a** $\arcsin x = \frac{\pi}{3}$ **b** $\arctan(x - 2) = \frac{\pi}{6}$

9 Show that $\sin\left(\frac{\pi}{8}\right) = \frac{1}{2}\sqrt{2 - \sqrt{2}}$ using a suitable double angle formula.

10 If α and β are the other angles of a right angled triangle, show that $\sin 2\alpha = \sin 2\beta$.

11 Prove that: **a** $(\sin\theta + \cos\theta)^2 = 1 + \sin 2\theta$ **b** $\csc 2x + \cot 2x = \cot x$

12 Use the principle of mathematical induction to prove that:

$$\sin^2\theta + \sin^2 2\theta + \sin^2 3\theta + ... + \sin^2(n\theta) = \frac{1}{2}\left[n - \frac{\cos[(n+1)\theta]\sin n\theta}{\sin\theta}\right] \quad \text{for all } n \in \mathbb{Z}^+.$$

13 **a** Show that $\cos(\alpha - \beta) - \cos(\alpha + \beta) = 2\sin\alpha\sin\beta$.

b Hence show that $\sin\alpha\sin\beta = \frac{1}{2}(\cos(\alpha - \beta) - \cos(\alpha + \beta))$.

c Hence show that $\sin[(k+1)\theta]\sin\frac{\theta}{2} + \sin\frac{k\theta}{2}\sin\frac{(k+1)\theta}{2} = \sin\frac{(k+1)\theta}{2}\sin\frac{(k+2)\theta}{2}$.

d Use the principle of mathematical induction to prove that:

$$\sin\theta + \sin 2\theta + \sin 3\theta + ... + \sin(n\theta) = \frac{\sin\left[\frac{1}{2}(n+1)\theta\right]\sin\left(\frac{1}{2}n\theta\right)}{\sin\left(\frac{1}{2}\theta\right)} \quad \text{for all } n \in \mathbb{Z}^+.$$

Section J, Part 1 (No Calculator)

1 If $f(x) = 7 + x - 3x^2$, find:

a $f(3)$ **b** $f'(3)$ **c** $f''(3)$.

2 Find $\frac{dy}{dx}$ for:

a $y = 3x^2 - x^4$ **b** $y = \frac{x^3 - x}{x^2}$

3 At what point on the curve $f(x) = \frac{x}{\sqrt{x^2+1}}$ does the tangent have gradient 1?

4 Find $\frac{dy}{dx}$ if:

a $y = e^{x^3+2}$ **b** $y = \ln\left(\frac{x+3}{x^2}\right)$ **c** $\ln(2y+1) = xe^y$

5 Given $y = 3e^x - e^{-x}$, show that $\frac{d^2y}{dx^2} = y$.

6 Differentiate with respect to x:

a $\sin(5x)\ln(x)$ **b** $\sin(x)\cos(2x)$ **c** $e^{-2x}\tan x$

7 Find the gradient of the tangent to $y = \sin^2 x$ at the point where $x = \frac{\pi}{3}$.

8 Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ given $x^2 + 2xy + y^2 = 4$.

9 Determine the derivative with respect to t of:

a $M = (t^2+3)^4$ **b** $A = \frac{\sqrt{t+5}}{t^2}$

10 Use the rules of differentiation to find $\frac{dy}{dx}$ for:

a $y = \frac{4}{\sqrt{x}} - 3x$ **b** $y = \sqrt{x^2 - 3x}$

11 Find $f''(2)$ for:

a $f(x) = 3x^2 - \frac{1}{x}$ **b** $f(x) = \sqrt{x}$

12 Given $y = (1 - \frac{1}{3}x)^3$, show that $\frac{d^3y}{dx^3} = -\frac{2}{9}$.

13 For $y = \frac{1}{2x+1}$, prove that $\frac{d^ny}{dx^n} = \frac{(-2)^n n!}{(2x+1)^{n+1}}$ for all $n \in \mathbb{Z}^+$.

Section J, Part 2 (Calculator)

1 Differentiate with respect to x:

a $5x - 3x^{-1}$ **b** $(3x^2 + x)^4$ **c** $(x^2 + 1)(1 - x^2)^3$

2 Find all points on the curve $y = 2x^3 + 3x^2 - 10x + 3$ where the gradient of the tangent is 2.

3 If $y = \sqrt{5 - 4x}$, find:

a $\frac{dy}{dx}$ **b** $\frac{d^2y}{dx^2}$ **c** $\frac{d^3y}{dx^3}$

4 Consider the curves $y = e^{x-1} + 1$ and $y = 3 - e^{1-x}$.

a Sketch the curves on the same set of axes.

b Find the point of intersection of the two curves.

c Show that the tangents to each curve at this point have the same gradient.

d Comment on the significance of this result.

5 Find $\frac{dy}{dx}$ if:

a $y = \ln(x^3 - 3x)$ **b** $y = \frac{e^x}{x^2}$ **c** $e^{x+y} = \ln(y^2 + 1)$

6 Find x if $f''(x) = 0$ and $f(x) = 2x^4 - 4x^3 - 9x^2 + 4x + 7$.

7 If $f(x) = x - \cos x$, find:

a $f(\pi)$ **b** $f'(\frac{\pi}{2})$ **c** $f''(\frac{3\pi}{4})$

8 Given that a and b are constants, differentiate $y = 3\sin bx - a\cos 2x$ with respect to x. Find a and b if $y + \frac{d^2y}{dx^2} = 6\cos 2x$.

9 Differentiate with respect to x:

a $10x - \sin(10x)$ **b** $\ln\left(\frac{1}{\cos x}\right)$ **c** $\sin(5x)\ln(2x)$

10 Differentiate with respect to x:

a $f(x) = \frac{(x+3)^3}{\sqrt{x}}$ **b** $f(x) = x^4\sqrt{x^2 + 3}$

11 Find $\frac{dy}{dx}$ for:

a $y = \frac{x}{\sqrt{\sec x}}$ **b** $y = e^x \cot(2x)$ **c** $y = \arccos\left(\frac{x}{2}\right)$

12 The curve $f(x) = 2x^3 + Ax + B$ has a tangent with gradient 10 at the point $(-2, 33)$. Find the values of A and B.

13 Find $\frac{dy}{dx}$ given $x^2 - 3y^2 = 0$. Explain your answer.

Section J, Part 3 (Calculator)

1 Differentiate with respect to x:

a $y = x^3\sqrt{1-x^2}$ **b** $y = \dfrac{x^2-3x}{\sqrt{x+1}}$

2 Find $\dfrac{d^2y}{dx^2}$ for:

a $y = 3x^4 - \dfrac{2}{x}$ **b** $y = x^3 - x + \dfrac{1}{\sqrt{x}}$

3 Find all points on the curve $y = xe^x$ where the gradient of the tangent is $2e$.

4 Differentiate with respect to x:

a $f(x) = \ln(e^x + 3)$ **b** $f(x) = \ln\left[\dfrac{(x+2)^3}{x}\right]$ **c** $f(x) = x^{x^2}$

5 Given $y = \left(x - \dfrac{1}{x}\right)^4$, find $\dfrac{dy}{dx}$ when $x = 1$.

6 **a** Find $f'(x)$ and $f''(x)$ for $f(x) = \sqrt{x}\cos(4x)$.

b Hence find $f'\left(\frac{\pi}{16}\right)$ and $f''\left(\frac{\pi}{8}\right)$.

7 Suppose $y = 3\sin 2x + 2\cos 2x$. Show that $4y + \dfrac{d^2y}{dx^2} = 0$.

8 Consider $f(x) = \dfrac{6x}{3+x^2}$. Find the value(s) of x when:

a $f(x) = -\frac{1}{2}$ **b** $f'(x) = 0$ **c** $f''(x) = 0$

9 The function f is defined by $f : x \mapsto -10\sin 2x\cos 2x$, $0 \leqslant x \leqslant \pi$.

a Write down an expression for $f(x)$ in the form $k\sin 4x$.

b Solve $f'(x) = 0$, giving exact answers.

10 Given the curve $e^x y - xy^2 = 1$, find:

a $\dfrac{dy}{dx}$ **b** the gradient of the curve at $x = 0$.

11 Prove using the principle of mathematical induction that if $y = x^n$, $n \in \mathbb{Z}^+$, then $\dfrac{dy}{dx} = nx^{n-1}$. You may assume the product rule of differentiation.

Section K , Part 1 (No Calculator)

1 Find the equation of the tangent to $y = -2x^2$ at the point where $x = -1$.

2 Find the equation of the normal to $y = \dfrac{1-2x}{x^2}$ at the point where $x = 1$.

3 Consider the function $f(x) = \dfrac{3x-2}{x+3}$.

- **a** State the equation of the vertical asymptote.
- **b** Find the axes intercepts.
- **c** Find $f'(x)$ and draw its sign diagram.
- **d** Does the function have any stationary points?

4 Find the equation of the normal to $y = e^{-x^2}$ at the point where $x = 1$.

5 Show that the equation of the tangent to $y = x\tan x$ at $x = \frac{\pi}{4}$ is $(2+\pi)x - 2y = \dfrac{\pi^2}{4}$.

6 The tangent to $y = \dfrac{ax+b}{\sqrt{x}}$ at $x = 1$ is $2x - y = 1$. Find a and b.

7 Show that the equation of the tangent to $f(x) = 4\ln(2x)$ at the point $\text{P}(1,\ 4\ln 2)$ is given by $y = 4x + 4\ln 2 - 4$.

8 Consider the function $f(x) = \dfrac{e^x}{x-1}$.

- **a** Find the y-intercept of the function.
- **b** For what values of x is $f(x)$ defined?
- **c** Find the signs of $f'(x)$ and $f''(x)$ and comment on the geometrical significance of each.
- **d** Sketch the graph of $y = f(x)$.
- **e** Find the equation of the tangent at the point where $x = 2$.

9 The line through $\text{A}(2,\ 4)$ and $\text{B}(0,\ 8)$ is a tangent to $y = \dfrac{a}{(x+2)^2}$. Find a.

10 Find the coordinates of P and Q if (PQ) is the tangent to $y = \dfrac{5}{\sqrt{x}}$ at $(1,\ 5)$.